In order to formulate a fatigue relation, the semirange in shear stress and the mean shear stress are needed. These stresses are defined as

$$S_r = \frac{S_{max} - S_{min}}{2} \qquad (5a, b)$$

$$S_m = \frac{S_{max} + S_{min}}{2}$$

respectively.

The Goodman fatigue relation in terms of shear stresses is assumed. This relation is

$$\frac{S_r}{S_e} + \frac{S_m}{S_u} = 1, \text{ for } S_m \geqq 0 ,$$

where S_e is the endurance limit in shear and S_u is the ultimate shear stress. For $S_u = 1/2\ \sigma_u$, where σ_u is the ultimate tensile stress, this relation can be rewritten as:

$$\frac{S_r}{S_e} + \frac{2S_m}{\sigma_u} = 1, \ S_m \geqq 0 \qquad (6)$$

The stresses S_r and S_m given by Equations (5a, b) can be calculated from elasticity solutions. In order to employ the fatigue relation (6) for general use, it is assumed that S_e can be related to S_u. This is a valid assumption as shown by Morrison, et al(10). From the data of Reference (10), it is found that the following relation between S_e and σ_u may be assumed:

$$S_e = \frac{1}{3}\sigma_u \qquad (7)$$

Substitution of Relation (7) into (6) gives

$$3S_r + 2S_m = \sigma_u \qquad (8)$$

For design purposes this equation can be made conservative by rewriting it as

$$3S_r + 2S_m = \sigma, \text{ where } \sigma \leqq \sigma_u \qquad (9)$$

Equation (9) now has a factor of safety, σ_u/σ and can be expected to predict lifetimes for 10^6 cycles and greater for ductile steels based upon the Goodman relation and available fatigue data. (Of course, stress concentration factors due to geometrical discontinuities or material flaws would reduce the expected lifetime.)

Fatigue Criterion for High-Strength Liner

Triaxial fatigue data on high-strength steels ($\sigma_u \geqq 250$ ksi) are not available. In fact, fatigue data of any sort are very limited. Therefore, a fatigue criterion for high-strength steels under triaxial fatigue cannot be as well established as it was for the lower strength steels. The high-strength steels are expected to fail in a brittle manner. Accordingly, a maximum tensile stress criterion of fatigue failure is postulated.

Because fatigue data are limited while tensile data are available, the tensile stresses $(\sigma)_r$ and $(\sigma)_m$ are assumed to be related to the ultimate tensile strength by two parameters α_r and α_m, which are defined as follows:

$$\alpha_r \equiv \frac{(\sigma)_r}{\sigma_1}, \; \alpha_m \equiv \frac{(\sigma)_m}{\sigma_1} \qquad (10a, b)$$

where $(\sigma)_r$ is the semirange in stress, $(\sigma)_m$ is the mean stress*, and σ_1 is less than or equal to the ultimate tensile strength depending upon the factor of safety desired. In order to get some estimations of what values α_r and α_m may be, some fatigue data from the literature on rotating-beam and push-pull tests are examined. References (11), (12), (13), and (14) give such fatigue data for 18% Ni maraging, H-11, D6AC, and Vascojet 1000 and other high-strength steels having ultimate tensile strengths of 250,000 to 310,000 psi at room temperature.

The fatigue life again is found to depend on the range in stress and the mean stress, and upon the temperature. This dependence is illustrated in Figure 3 for 10^4 to 10^5 cycles life in terms of the parameters α_r and α_m. The 1000 F temperature data are for Vascojet 1000. Although α_r increases with temperature for this steel, the ultimate tensile strength decreases and the fatigue strength at 10^4 to 10^5 cycles for $\alpha_m = 0$ remains nearly constant over the temperature range of 75 F to 1000 F.

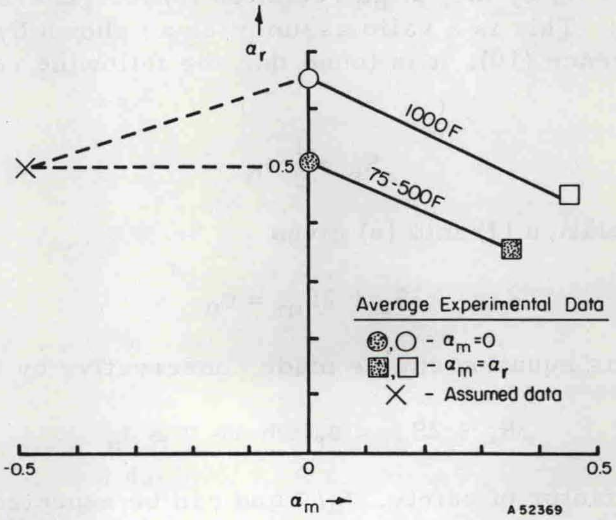

FIGURE 3. FATIGUE DIAGRAM FOR 10^4-10^5 CYCLES LIFE FOR HIGH-STRENGTH STEELS AT TEMPERATURES OF 75 F – 1000 F

α_r and α_m are defined by Equations (10a, b)

The fatigue data available are only for positive and zero mean stresses. However, there is evidence that compressive mean stress may significantly increase the fatigue strength(15, 16). The reasons for this are thought to be that compression may reduce the detrimental effect of fluid pressure entering minute cracks or voids in the material and the compression may restrain such flaws from growing. Since the liner of a high-

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*σ_r and σ_m are defined by expressions similar to Equations (5a, b) for S_r and S_m.